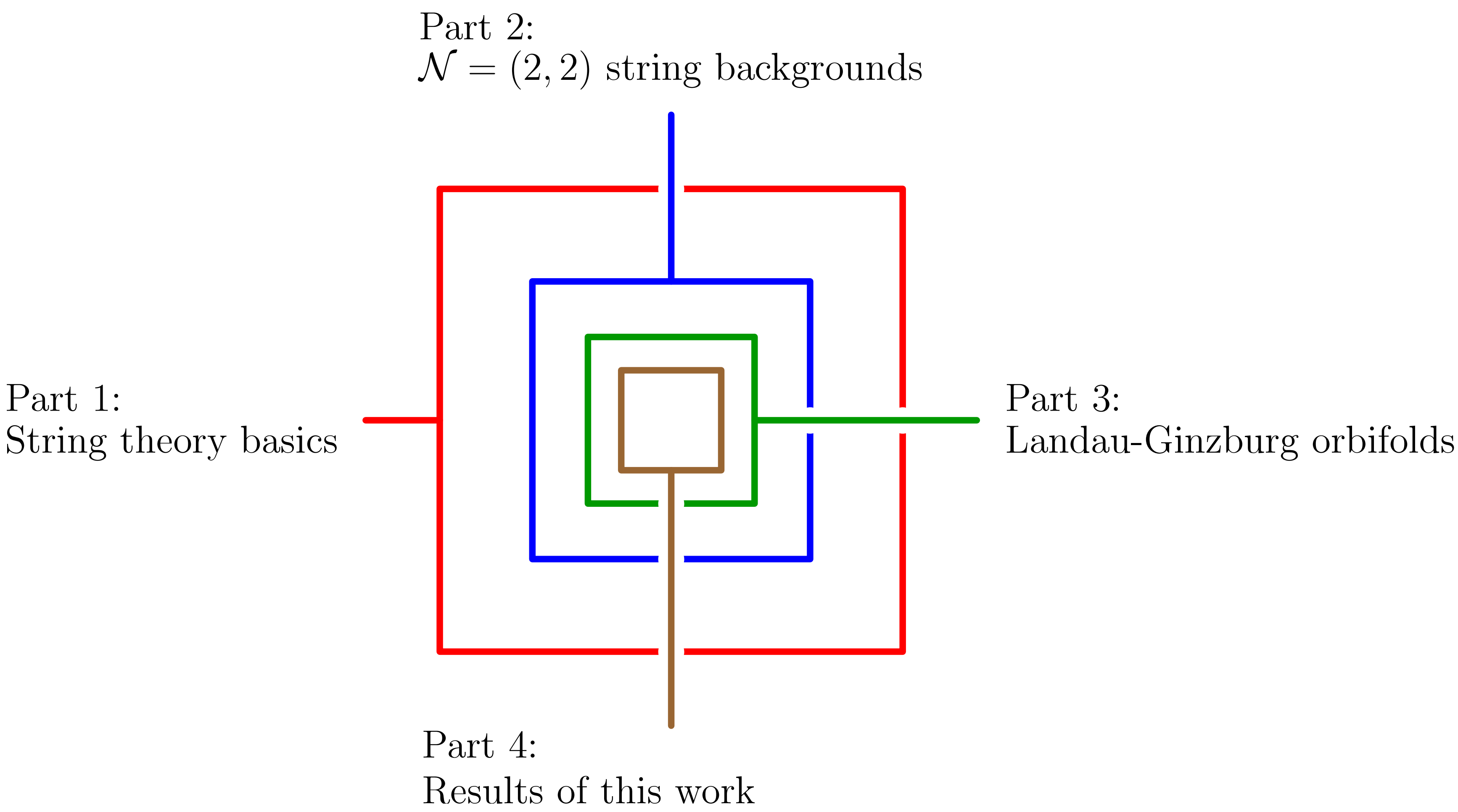
Master Thesis
Landau-Ginzburg String Backgrounds with Orientifolds and D-branes

Author: Samo Jordan

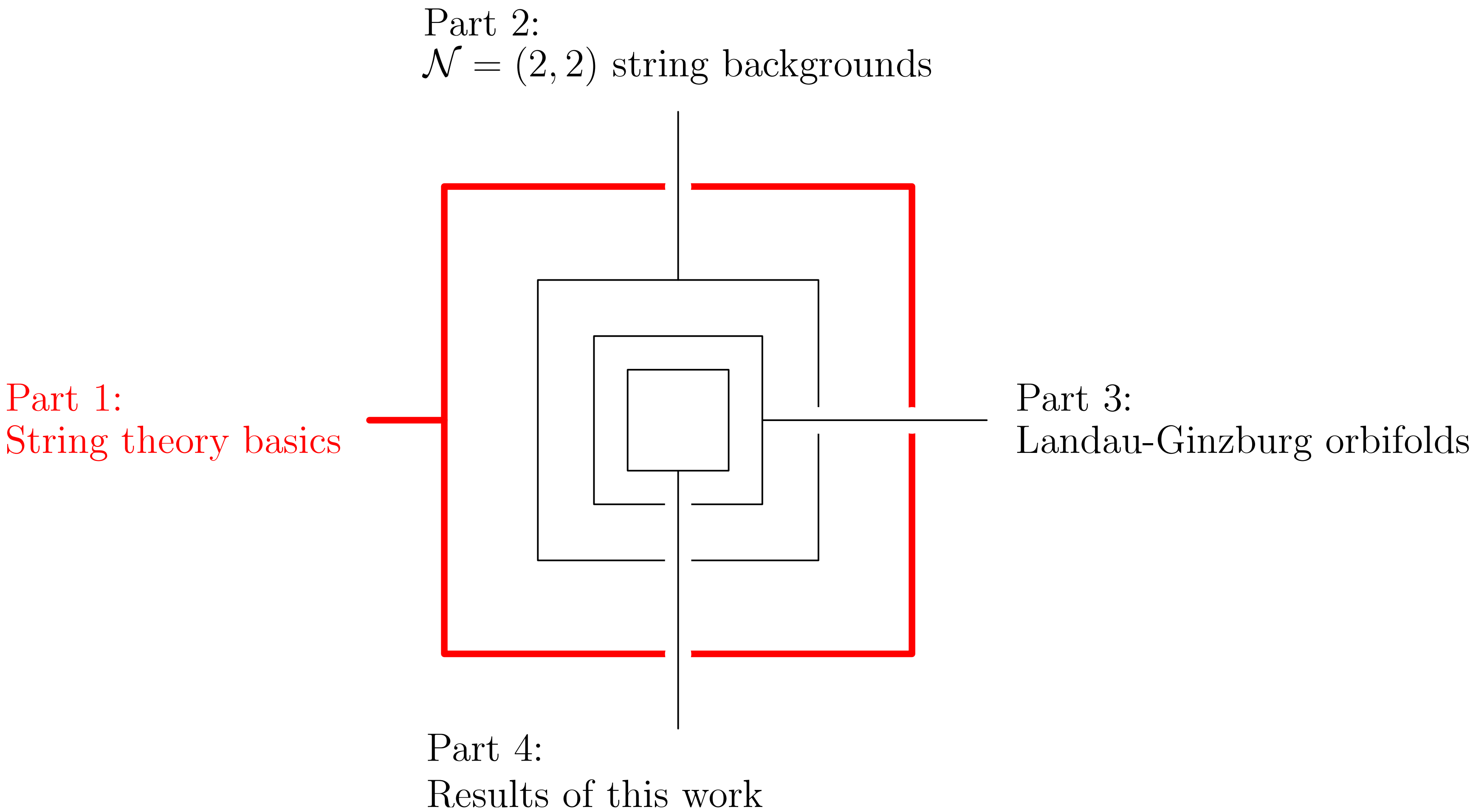
Supervisor: Prof. Ilka Brunner

Institute of Theoretical Physics, University of Zürich

Contents

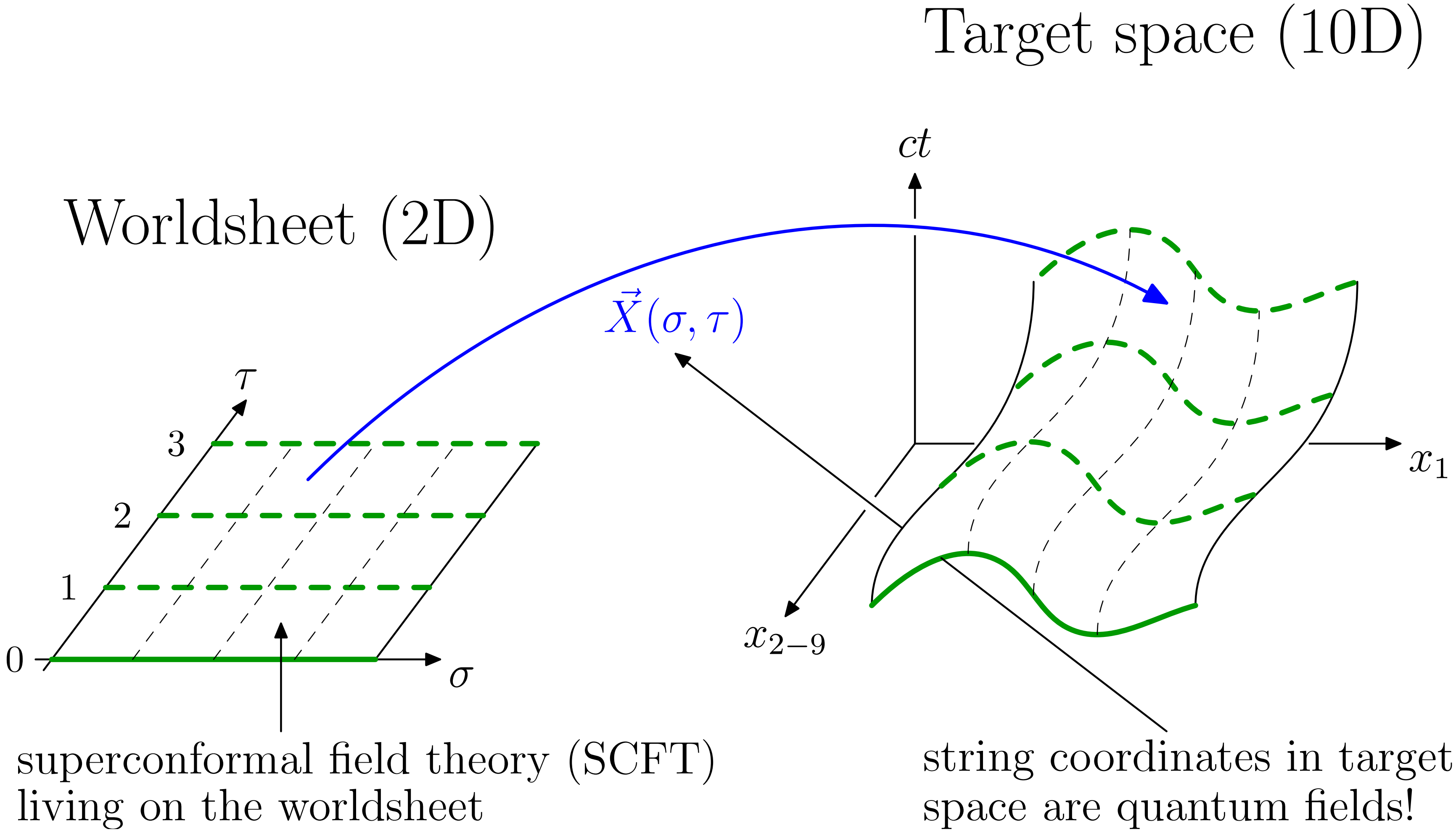

Part 2:
$\mathcal{N} = (2,2)$ string backgrounds
Part 1:
String theory basics
Part 3:
Landau-Ginzburg orbifolds
Part 4:
Results of this work

Part 1



Part 2:
$\mathcal{N} = (2,2)$ string backgrounds
Part 1:
String theory basics
Part 3:
Landau-Ginzburg orbifolds
Part 4:
Results of this work

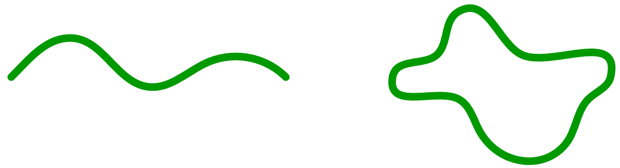
Introducing the string



The string theory zoo

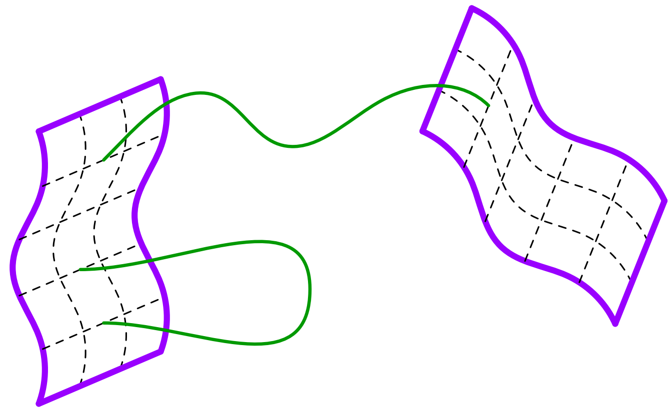
Strings:

- open and closed strings
- perturbative, dynamical objects



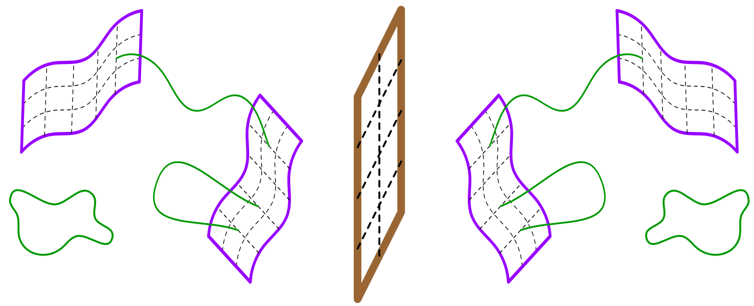
D-branes:

- nonperturbative, dynamical objects
- perturbative description as locus of open string endpoints



Orientifolds:

- perturbative, nondynamical objects
- implement parity symmetry



String backgrounds

String theory is defined as a perturbative expansion ("sum over world sheets") around a string vacuum = string background.

String backgrounds are characterized by

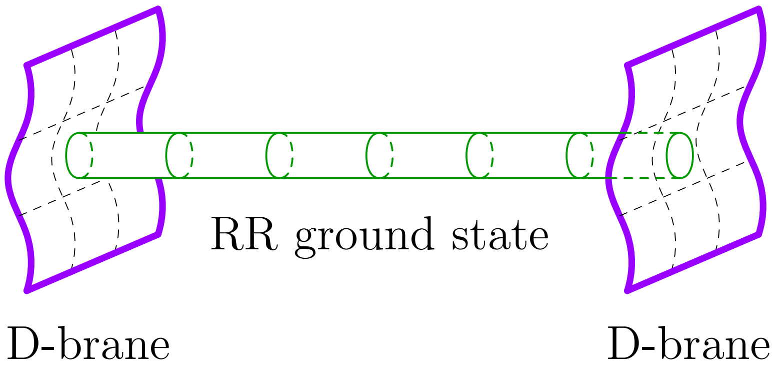
- target space geometry
- D-brane configuration
- Orientifold configuration
- other things not important for this work

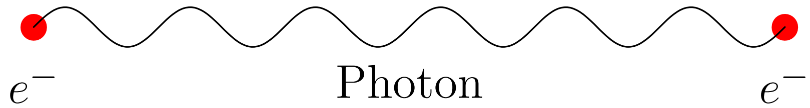
The 10 target space dimensions split up into

- 4 external dimensions (extended)
- 6 internal dimensions (compactified, i.e. Calabi-Yau manifold)

The tadpole condition

D-branes and Orientifolds are charged under Ramond-Ramond ground states. These closed strings mediate the force in a similar way as photons do for charged point particles:





A nonvanishing total charge leads to divergent tadpole amplitudes which render the string background inconsistent (tadpole anomaly)!

$\Rightarrow$ The charges (tadpoles) of all D-branes and Orientifolds in our string background need to sum up to zero!

Supersymmetry

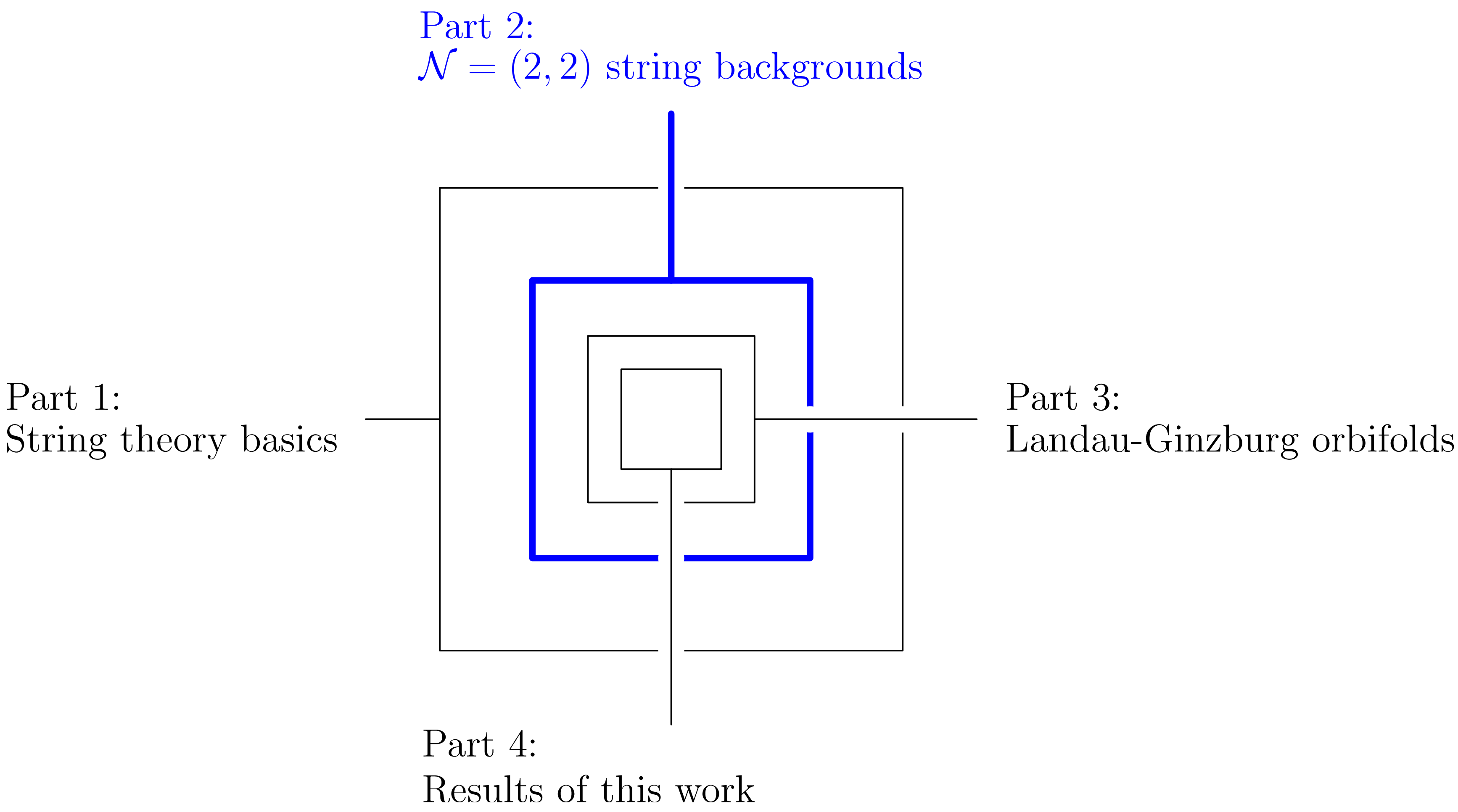
In string theory we can have two types of supersymmetry:

- worldsheet supersymmetry
  - required to eliminate tachyons from the string spectrum
- spacetime supersymmetry:
  - required to connect string theory to the supersymmetric standard model

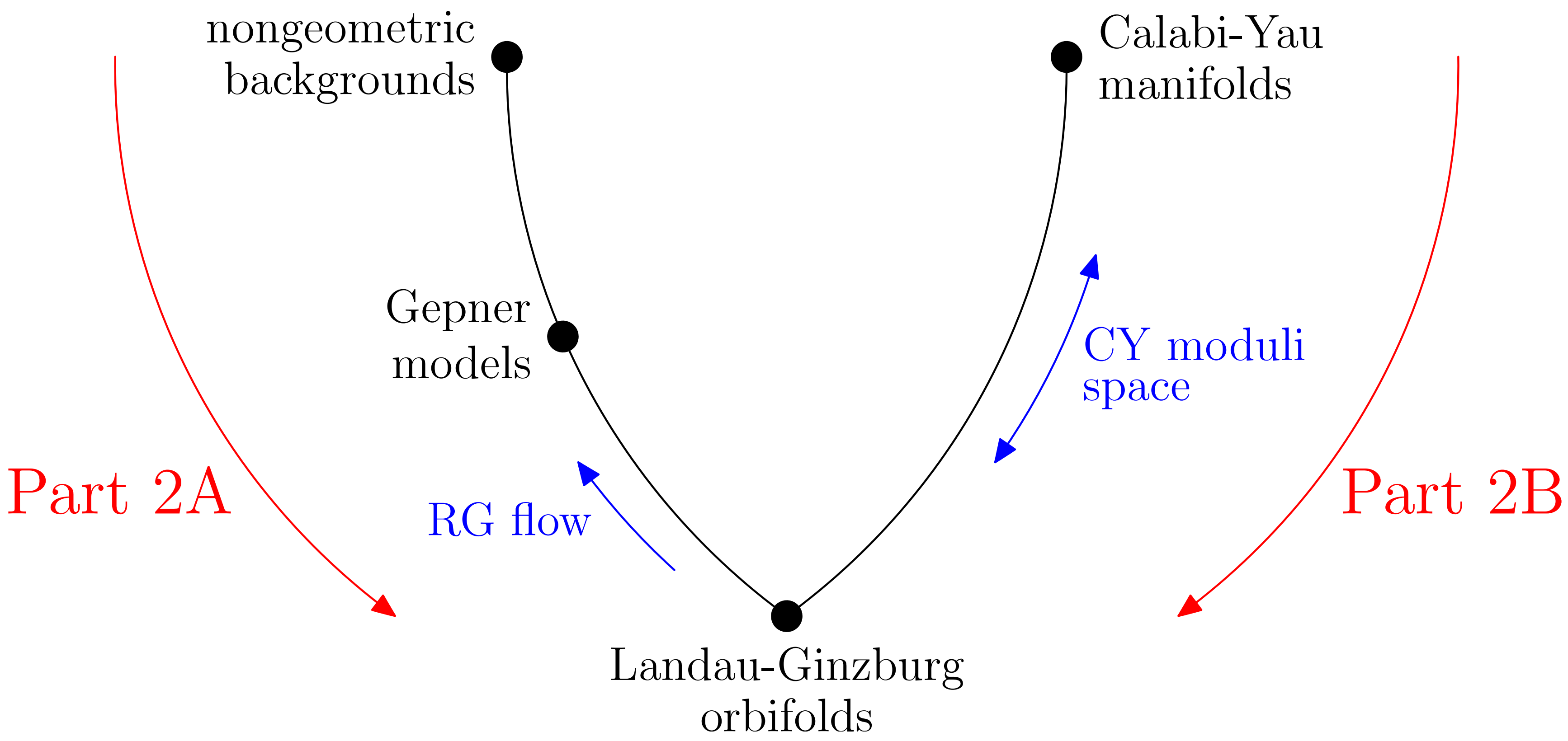
In the following we will focus on string backgrounds being

- tadpolefree
- supersymmetric both on the worldsheet and in spacetime

Part 2

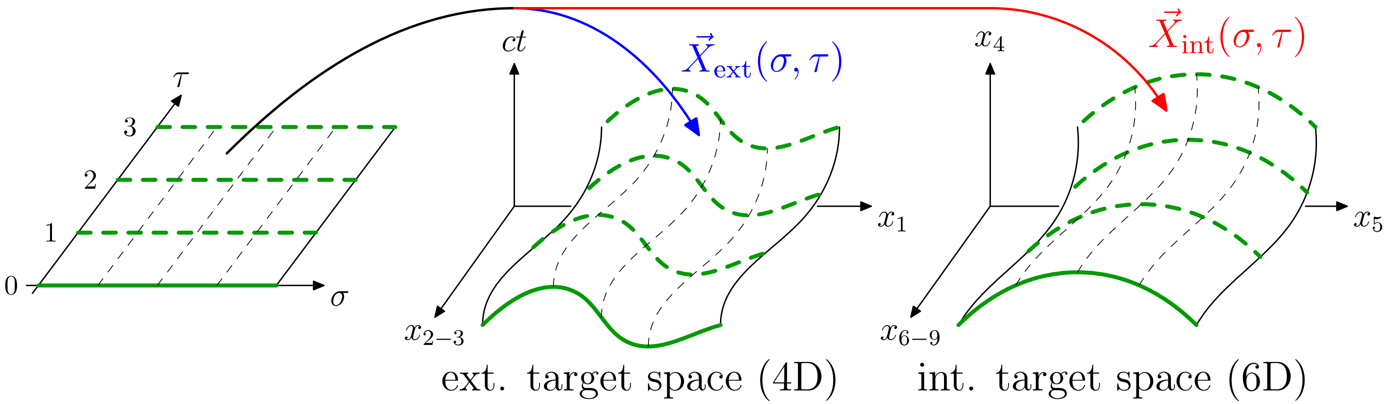

Part 2:
$\mathcal{N} = (2,2)$ string backgrounds
Part 1:
String theory basics
Part 3:
Landau-Ginzburg orbifolds
Part 4:
Results of this work

Two paths to Landau-Ginzburg orbifolds

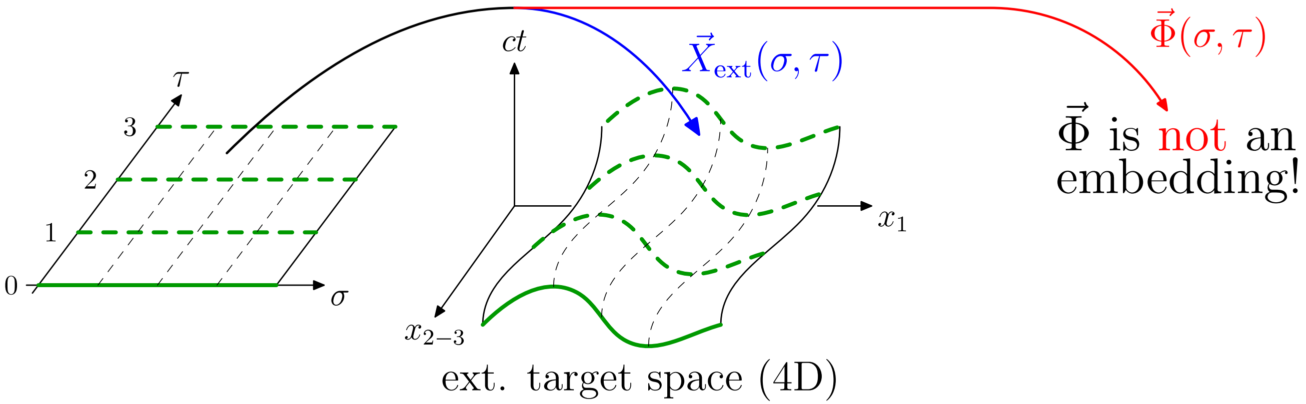


Nongeometric string backgrounds

geometric background



nongeometric background



Gepner models

The 'internal' string propagation can be described by any unitary SCFT. We are interested in $\mathcal{N} = (2,2)$ SCFT's with central charge $c = 9$.

The simplest $\mathcal{N} = (2,2)$ SCFT's are the Minimal models M_k with central charge $c = 3k/(k+2), k = 1, 2, \ldots$ There is no way in achieving $c = 9$ with one Minimal model!

By tensoring together several Minimal models $c = 9$ is possible! Example:

$$M_3 \otimes M_3 \otimes M_3 \otimes M_3 \otimes M_3 \quad \Rightarrow \quad c = 5 * \frac{9}{5} = 9$$

Applying an orbifolding procedure yields the Gepner models, which qualify as spacetime-supersymmetric nongeometric string backgrounds.

Landau-Ginzburg orbifolds

A Landau-Ginzburg theory is a $\mathcal{N} = (2,2)$ supersymmetric (but not conformal) field theory described by the action

$$S_{LG} = \int d^2z d^4\theta K(\Phi_i, \overline{\Phi}_i) + \left(\int d^2z d^2\theta W(\Phi_i) + \text{c.c.} \right)$$

Φ_i are chiral superfields and $W(\Phi_i)$ is called the superpotential.

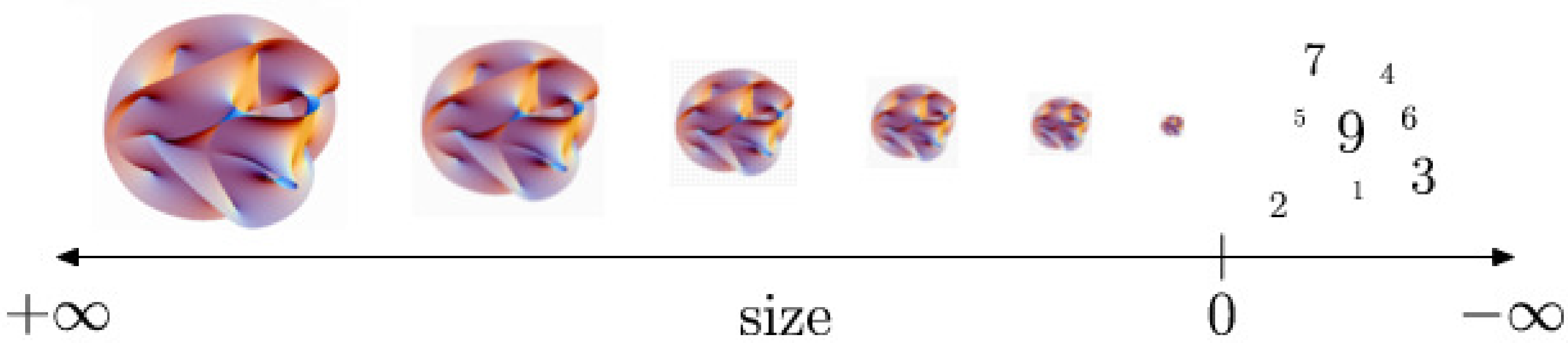
Let $\mathbb{Z}_H \ni g : \Phi_i \mapsto g\Phi_i$ be a symmetry of the LG action. The associated quotient theory is called Landau-Ginzburg orbifold.

Certain Landau-Ginzburg orbifolds flow into Gepner models by the renormalization group. The superpotential is not renormalized and encodes important data of the Gepner model. Thus LG orbifolds are useful to study nongeometric string backgrounds!

The Calabi-Yau moduli space

For geometric string backgrounds, the internal target space is typically a Calabi-Yau manifold (= compact Ricci-flat Kähler manifold). Their size and shape is described by a set of parameters called moduli. These span the Calabi-Yau moduli space.

By continuation the 'size moduli' can even become negative! In these regions the string backgrounds become nongeometric.

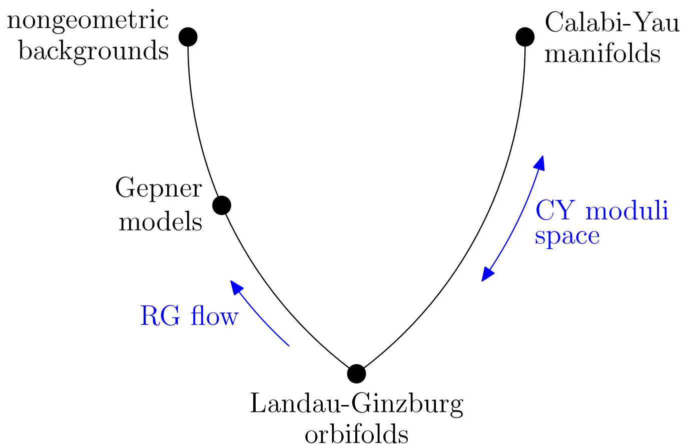


The LG-CY correspondence

For some Calabi-Yau manifolds, the continuation to 'negative size' leads to Landau-Ginzburg orbifold theories!

Landau-Ginzburg orbifolds are thus suitable to study

- those properties of nongeometric string backgrounds, which are stable under the RG flow
- those properties of Calabi-Yau string backgrounds, which are stable under continuation through the moduli space



Breaking spacetime-supersymmetry

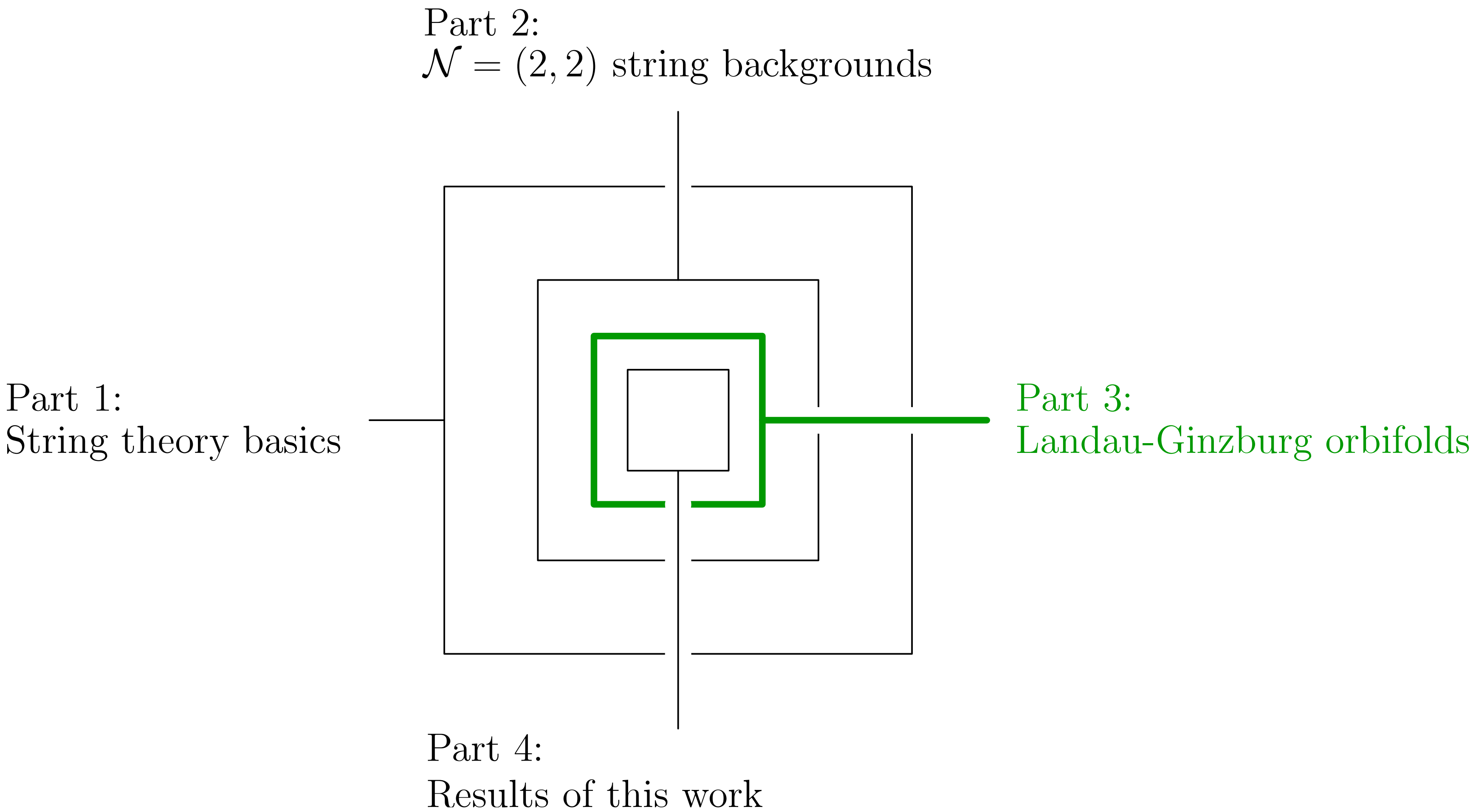
The string backgrounds described by Landau-Ginzburg orbifolds have $\mathcal{N} = 2$ spacetime-supersymmetry! To build phenomelogically interesting backgrounds we want to break half of the supersymmetry. This can be achieved by adding D-branes/Orientifolds!

Let us formulate the aim of this work:

Build Landau-Ginzburg string backgrounds with Orientifolds and D-branes

- satisfying the tadpole condition
- preserving $\mathcal{N} = 1$ spacetime-supersymmetry

Part 3


Part 2:
$\mathcal{N} = (2,2)$ string backgrounds
Part 1:
String theory basics
Part 3:
Landau-Ginzburg orbifolds
Part 4:
Results of this work

Topological LG orbifolds

In order to handle LG orbifolds with Orientifolds and D-branes we need to simplify.

A LG orbifold theory can be twisted into a topological field theory. It can be interpreted as describing a topological sector of the untwisted theory. This is essentially the restriction to the string ground states.

There exist two types of twisting: A-Twist and B-Twist. B-type topological LG orbifolds have the following virtues:

- The charges of D-branes and Orientifolds can be calculated with little effort
- D-branes and Orientifolds can be described in a categorical language (see later).

A global view of $\mathcal{N} = (2, 2)$ theories

The Gepner model and the Calabi-Yau compactification theory can also be twisted, leading to the following global picture:

$$\begin{tikzcd}
& \boxed{\text{Calabi-Yau}} \arrow[rr, dashed, "\text{Top. Twist}"] & & \boxed{\text{Top. Calabi-Yau}} \\
\boxed{\text{LG orbifold}} \arrow[ur, leftrightarrow, "\text{moduli space}"] \arrow[rr, dashed, "\text{Top. Twist}"] \arrow[d, "\text{RG flow}"'] & & \boxed{\text{Top. LG orbifold}} \arrow[ur, leftrightarrow, "\text{moduli space}"] \arrow[d, "\text{RG flow}"'] & \\
\boxed{\text{Gepner model}} \arrow[rr, dashed, "\text{Top. Twist}"] & & \boxed{\text{Top. Gepner model}} &
\end{tikzcd}$$

LG orbifolds of Fermat type

Recall the Landau-Ginzburg action:

$$S_{LG} = \int d^2 z d^4 \theta K(\Phi_i, \overline{\Phi}_i) + \left(\int d^2 z d^2 \theta W(\Phi_i) + \text{c.c.} \right)$$

The superpotential $W(\Phi_i)$ satisfies

$$W(\lambda^{w_i} \Phi_i) = \lambda^H W(\Phi_i) \quad \forall \lambda \in \mathbb{C}, \qquad w_i, H \in \mathbb{Z}$$

w_i are called the weights of the chiral superfields Φ_i. We restrict ourselves to superpotentials of Fermat type with $i = 1, \ldots, 5$:

$$W = \sum_{i=1}^{5} \Phi_i^{H/w_i}, \quad H = \sum_{i=1}^{5} w_i$$

There exist 147 five-variable Fermat models denoted by $\mathbb{P}_{(w_1,\ldots,w_5)}[H]$!

D-branes in LG-orbifolds

A (topological) D-brane is described by a quadruple (C, ρ, Q, γ):

- C is a $\mathbb{Z}_2$-graded $\mathbb{C}[\Phi_i]$-module
- ρ is a grading operator (i.e. $\rho = \mathrm{diag}(1, -1)$)
- $Q(\Phi_i)$ is a matrix factorization of the superpotential $W(\Phi_i)$:

$$Q(\Phi_i) = \begin{pmatrix} 0 & J(\Phi_i) \\ E(\Phi_i) & 0 \end{pmatrix}, \quad Q^2(\Phi_i) = W(\Phi_i) \cdot \mathbf{1}$$

- γ is a $\mathbb{Z}_H$-representation satisfying

$$\gamma Q(\omega^{w_i}\Phi_i)\gamma^{-1} = Q(\Phi_i), \quad \gamma^H = \mathbf{1}$$

The open strings stretching between two D-branes are roughly homomorphisms between the modules of both D-branes. More precisely they are determined by the cohomology of a nilpotent operator.

Orientifolds in LG-orbifolds

Topological D-branes are objects in a (triangulated) category! The open strings are the morphisms in this category.

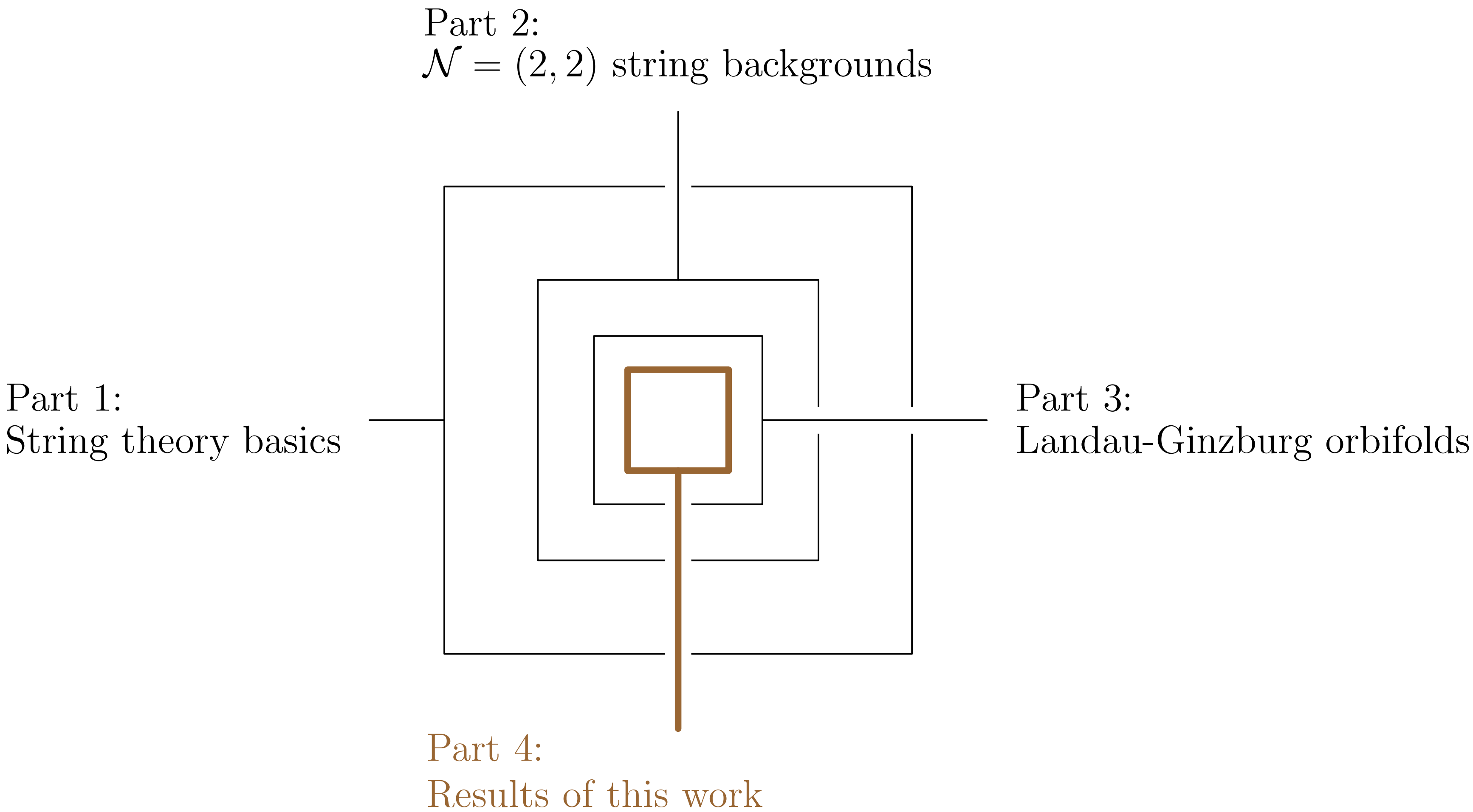
Orientifolds are implemented as functors:

$$\mathcal{P}(\tau) \ : \ (C, \rho, Q(\Phi_i), \gamma) \ \mapsto \ (C^*, \pm\rho^T, -Q(\tau\Phi_i)^T, \gamma^{-T})$$

Here $(\,\cdot\,)^T$ denotes the graded transpose and τ is the parity action on the chiral superfields Φ_i.

In an Orientifold theory all D-branes need to be parity-invariant! An important part of this work is thus to work out the conditions for D-branes to be invariant under a given parity.

Part 4


Part 2:
$\mathcal{N} = (2, 2)$ string backgrounds
Part 1:
String theory basics
Part 3:
Landau-Ginzburg orbifolds
Part 4:
Results of this work

The D-brane charge formula

D-branes couple to Ramond-Ramond ground states. A method of finding all RR ground states has been worked out in the Thesis. Furthermore a formula to calculate D-brane charges has been derived:

$$\mathrm{ch}(Q,\gamma)(|\phi\rangle^l) = R^l_Q(t_a)\cdot\omega^{pl}\prod_{i\in I_T\cap I_{l,t}}(1-\omega^{w_i n_i l})\prod_{i\in I_P\cap I_{l,t}}(1-\omega^{\widetilde{w}_i|\mathcal{I}_i|l})$$

$$R^l_Q(t_a) = \begin{cases} 1 & \text{if } |I_{l,t}|=5 \\ 0 & \text{if } |I_{l,t}|=3 \text{ and } a,b\in I_T \\ \frac{\widetilde{w}_a}{H}\sum_{i\in\mathcal{I}_a}\omega^{w_a(-i-1/2)(t_a+1)} & \text{if } |I_{l,t}|=3 \text{ and } a,b\in\widetilde{I}_P \end{cases}$$

$$|\phi\rangle^l = \begin{cases} |0\rangle^l_{(R,R)} & \text{if } |I_{l,t}|=5 \\ x_a^{t_a}x_b^{t_b}|0\rangle^l_{(R,R)} & \text{if } |I_{l,t}|=3 \end{cases}$$

The Orientifold charge formula

$$\mathrm{ch}(\mathcal{P}(\tau))(|\phi\rangle^l) = \epsilon\omega^{\frac{-3M\chi l}{2}} R^l_P(t_a) \prod_{i\in I_\sigma\cap I_{l,t}} (1-\omega^{w_i l}) \prod_{i\in I_{\mathrm{id}}\cap I_{l,t}} (1-\omega^{(\frac{w_i+H}{2})l})$$

if H odd

$$\mathrm{ch}(\mathcal{P}(\tau))(|\phi\rangle^l) = \epsilon\omega^{\frac{-3M\chi l}{2}} R^l_P(t_a) \prod_{i\in I_\sigma\cap I_{l,t}} (1-\omega^{w_i l}) \prod_{i\in I_{\mathrm{id},E}\cap I_{l,t}} (1+\omega^{w_i(m_i+\frac{l}{2})})$$

$$\times \Big(\prod_{i\in I_{\mathrm{id},O}\cap I_{l,t}} (1+\omega^{w_i(m_i+\frac{l}{2})}) + \prod_{i\in I_{\mathrm{id},O}\cap I_{l,t}} (1-\omega^{w_i(m_i+\frac{l}{2})}) \Big)$$

if H even and l odd

$$\mathrm{ch}(\mathcal{P}(\tau))(|\phi\rangle^l) = 0 \qquad \text{if H even and l even}$$

$$R^l_P(t_a) = \begin{cases} 1 & \text{if } |I_{l,t}| = 5 \\ 0 & \text{if } |I_{l,t}| = 3 \text{ and } a,b\in I_{\mathrm{id}} \\ \frac{w_a}{H}\omega^{w_a(h_a/2+m_a)(t_a+1)} & \text{if } |I_{l,t}| = 3 \text{ and } a,b\in \widetilde{I}_\sigma \end{cases}$$

Parity invariance conditions

All D-branes and Orientifolds in the models considered can be split into elementary building blocks. The associated parity invariance conditions have been worked out:

Mat. fac. & Parity	bosonic invariance	fermionic invariance	bos. & ferm. invariance
TP $\sigma = \mathrm{id}$	$n = h/2$	always	$n = h/2$
$\mathrm{TP} \otimes \mathrm{TP}$ $\sigma \neq \mathrm{id}$	never	never	never
Perm $\sigma = \mathrm{id}$	$\eta_j \in \mathcal{I} \;\Leftrightarrow\; \eta_{j_*} \in D\backslash\mathcal{I}$ $j_* = j + m_1 - m_2 \quad \text{mod d}$	$\eta_j \in \mathcal{I} \;\Leftrightarrow\; \eta_{j_*} \in \mathcal{I}$	never
Perm $\sigma \neq \mathrm{id}$	$\eta_j \in \mathcal{I} \;\Leftrightarrow\; \eta_{j_*} \in D\backslash\mathcal{I}$ $j_* = h - 1 - (j + m_1 - m_2) \quad \text{mod h}$	$\eta_j \in \mathcal{I} \;\Leftrightarrow\; \eta_{j_*} \in \mathcal{I}$	never

Using the information in this table, the parity invariance conditions for D-branes and Orientifolds containing these building blocks have been derived.

More results

Some more results worked out in the Thesis:

- Conditions for spacetime-supersymmetry in the presence of D-branes and Orientifolds.
- Formula to calculate the D-brane gauge groups.
- Comparison with CFT-based results. In some cases disagreement was found and suggestions concerning the source of these disagreements have been written down in the Thesis.
- All 'simple' tadpolefree and spacetime-supersymmetric backgrounds are presented (see next slide).
- A general algorithmic method of how to find more general backgrounds is shown.

Simple tadpolefree backgrounds

By visually analyzing the charge formulas one finds four configurations solving the tadpole condition and which are spacetime-supersymmetric:

Model	4 D-branes	Orientifold, $\mathcal{P}(\tau) = \mathscr{A}\mathscr{P}(\tau)$, $\epsilon = -1,\ M_\chi = 0,\ m_i = 0\ \forall i$
$\mathbb{P}_{(1,1,1,1,1)}[5]$	$M = 5,\ (3,3,3,3,3)$ $M = 5,\ (3,\{\eta_2\},\{\eta_2\})$	$\sigma = \mathrm{id}$ $\sigma = (2 \leftrightarrow 3, 4 \leftrightarrow 5)$
$\mathbb{P}_{(1,1,1,3,3)}[9]$	$M = 9,\ (5,\{\eta_4\},\{\eta_1\})$	$\sigma = (2 \leftrightarrow 3, 4 \leftrightarrow 5)$
$\mathbb{P}_{(1,3,3,7,7)}[21]$	$M = 21,\ (11,\{\eta_3\},\{\eta_1\})$	$\sigma = (2 \leftrightarrow 3, 4 \leftrightarrow 5)$